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Six Flavors of Quarks but only Three Flavors of Leptons[1]

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Fryberger's self-consistency condition for the mean leptonic mass in QED with bare-mass zero and a quantum-gravitational cutoff indicates that there exist six flavors of quarks but only three flavors of leptons. For three flavors there is an explicit expression for the charged lepton mass operator.

Recently Fryberger (1979) has estimated the mean mass $\bar{m}$ of the charged leptons from QED with bare-mass zero and a quantum-gravitational cutoff, along the lines initiated some years ago by the present author (Rosen, 1971). This approach to the self-mass problem derives its inspiration and impetus from the early proposal of Weisskopf (1939) and the subsequent results of Landau and co-workers (1956). Fryberger's self-consistency condition for $\bar{m}$ [given by his equation (27) with $R+2$ replaced here by $N+2\hat{N}$] is

$$(2\alpha/3\pi)\ln(M/\bar{m}) = (N+2\hat{N})^{-1}\{1-\exp[-4A(N+2\hat{N})/9]\} \tag{1}$$

in which $\alpha=(137.03596)^{-1}$ is the fine-structure constant, $M=(\hbar c/16\pi G)^{1/2}=1.72215\times 10^{21}$ MeV is the most probable quantum-gravitational mass cutoff (Delbourgo, 1969; Isham et al., 1971; Rosen, 1971) and $N,\hat{N}$ are the number of lepton flavors and (tricolored) quark flavors, respectively. The precise value of the constant parameter A in (1) requires higher-order approximation, but it can be asserted with reasonable certainty that $A=1.000\pm 0.400$ (see Figure 10 in Fryberger, 1979) for $N+2\hat{N}\geqslant 13$. What is particularly noteworthy regarding condition (1) is that it is relatively insensitive to higher-order approximation (e.g., see Figure 9 Fryberger, 1979) and the exact value of A; for $(N+2\hat{N})\geqslant 13$ and $A\geqslant 3/5$, the curly-bracketed factor in (1) is $(0.98439)\pm(0.01561)$.

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Now the mass values for the low-lying charged leptons are given by the semitheoretical formula (Nambu, 1952; Rosen, 1964; Barut, 1979)

$$m_n = m_e\left(1 + \tfrac{3}{2}\alpha^{-1}\gamma_n\right) \tag{2}$$

where

$$\gamma_1 = 0, \qquad \gamma_2 = 1, \qquad \gamma_3 = 17 \tag{3}$$

with the sequential labeling: $m_1 \equiv m_e, m_2 \equiv m_\mu, m_3 \equiv m_\tau$. The algorithm proposed by Barut (1979) for generating (3) is equivalent to the recurrence relation

$$\gamma_{n+1} = \gamma_n + n^4 \tag{4}$$

which gives[2]

$$\gamma_4 = 98, \qquad \gamma_5 = 354, \qquad \gamma_6 = 979 \tag{5}$$

From the assumption that (2) and (5) hold for $n \leqslant N \leqslant 6$, it follows that the mean lepton mass is

$$\bar{m} \equiv \frac{1}{N}\sum_{n=1}^{N} m_n = \begin{cases} 630.74 \text{ MeV} & \text{for } N=3 \\ 3046.64 \text{ MeV} & \text{for } N=4 \\ 9874.15 \text{ MeV} & \text{for } N=5 \\ 25{,}367.88 \text{ MeV} & \text{for } N=6 \end{cases} \tag{6}$$

and hence the left side of (1) becomes

$$(2\alpha/3\pi)\ln(M/\bar{m}) = \begin{cases} 0.0657 \text{ for } N=3 \\ 0.0633 \text{ for } N=4 \\ 0.0615 \text{ for } N=5 \\ 0.0600 \text{ for } N=6 \end{cases} \tag{7}$$

Thus, owing to the limited range of possible values for the curly-bracketed factor, (1) requires

$$N + 2\hat{N} = \begin{cases} 14.98 \pm 0.24 & \text{for } N=3 \\ 15.55 \pm 0.25 & \text{for } N=4 \\ 16.01 \pm 0.25 & \text{for } N=5 \\ 16.41 \pm 0.26 & \text{for } N=6 \end{cases} \tag{8}$$

[2]The general expression is rather complicated, namely,

$$\gamma_n = \sum_{k=1}^{n-1} k^4 = \frac{1}{5}n^5 - \frac{1}{2}n^4 + \frac{1}{3}n^3 - \frac{1}{30}n$$

Since their associated $\hat{N}$ values are nonintegral, the latter possibilities $N=4$, 5, or 6 are ruled out by (8). Only $N=3$ is admissible, with $\hat{N}=6$.

Although it cannot as yet be precluded with certainty, the heavy lepton predicted by (2) and (5) for $n=4$ (i.e., $m_4=10.29$ GeV) has not shown up in measurements of the $e^+ - e^-$ total cross section (Perl, 1979). For $N=3$ the quantum number label n in (2) can be dropped, along with relations (4) and (5), in favor of an operator T_3 with eigenvalues $+1, 0, -1$ and eigenstates identified with the physical particles:

$$T_3|e^\pm\rangle=0, \qquad T_3|\mu^\pm\rangle=-|\mu^\pm\rangle, \qquad T_3|\tau^\pm\rangle=|\tau^\pm\rangle \tag{9}$$

Then the mass operator which yields (2), (3) is given directly by

$$m=m_e\left\{1+\tfrac{3}{2}\alpha^{-1}\left[1+(T_3+1)^4\right]T_3^2\right\} \tag{10}$$

a form somewhat suggestive of Gell-Mann–Okubo splitting, where T_3 may be interpreted as the third component of "leptospin." The precise value of the constant A in (1) now follows from $\bar{m}=m_e(1+9\alpha^{-1})=630.7435$ MeV as $A=0.6408901$, which in turn fixes theoretical m_e in (10).

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